Industrial Engineering 546 Spring 2017
Economic Decisions in Engineering

Exam 3

Student Information (print neatly)

Name: ________________________________

Purdue email: ____________________________

Directions:

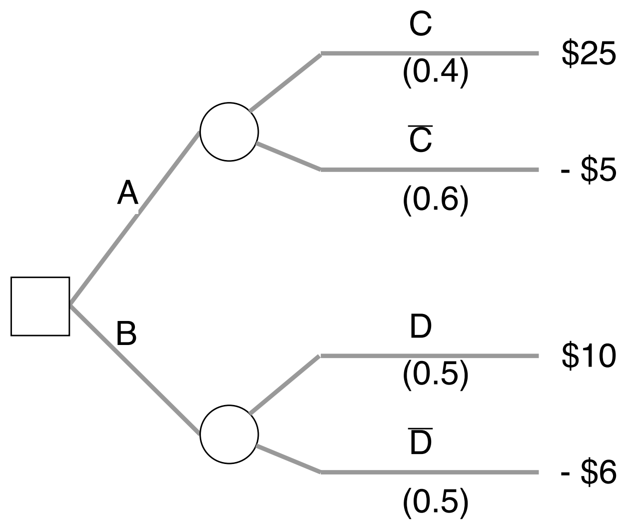
- You have 120 minutes to complete this exam. If you open the test before stated or do not turn it in on time you will lose 20 points.
- This exam is closed book and notes. You will receive a zero for this exam for using books, notes, electronic devices (cell phone, ipad, calculator, laptop, etc.). You will also be reported and appropriate disciplinary action will be taken.
- Your answers must be legible. Circle, underline, or leave sufficient white-space to distinguish your answers from intermediate work. The last two pages are scratch paper and may be torn out.
- Show all your work.
- There are 115 points total. The exam will be graded out of 100, so there are 15 bonus points. However, the last two questions will have limited partial credit.

Grade:

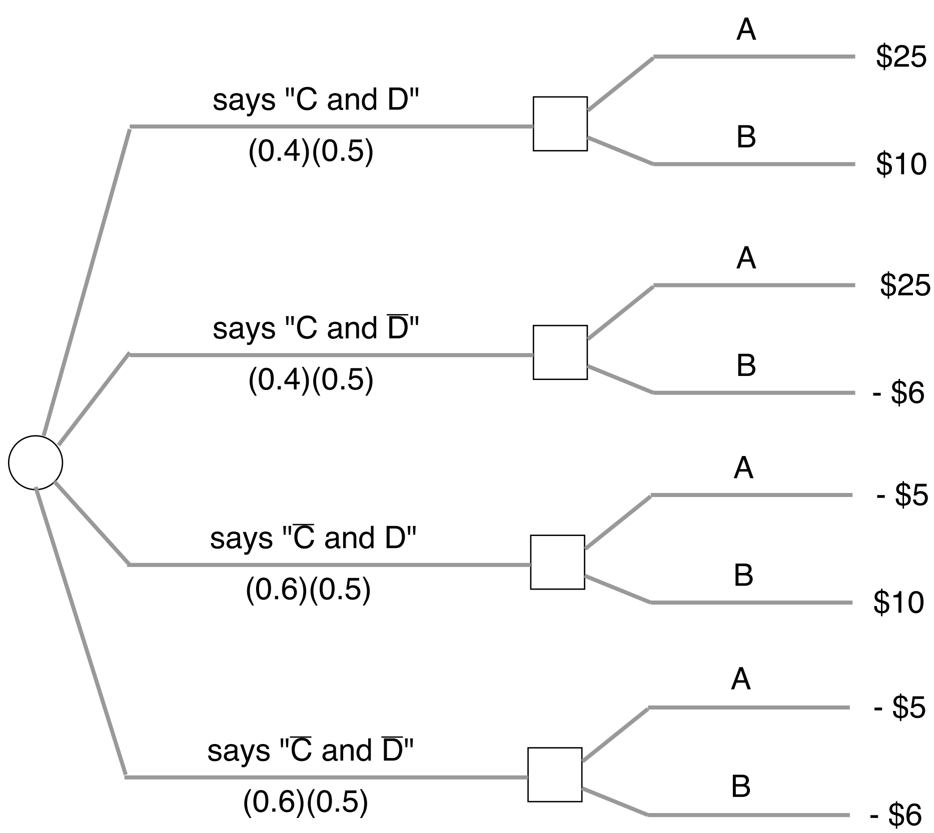
1. [35] ____________________________
2. [35] ____________________________
3. [25] ____________________________
4. [20] ____________________________

Total: ____________________________

Problem 1. [35 points] This problem deals with the value of information. Do not calculate. Leave terms like "$0.8 * 0.3$" as a product. For reference, E[earnings A] > E[earnings B].



A. Consider the decision tree above. Suppose you consult a perfect expert who will tell you the outcomes of both events. The events are independent. Draw the new decision tree where you use this information. Fully label the tree.



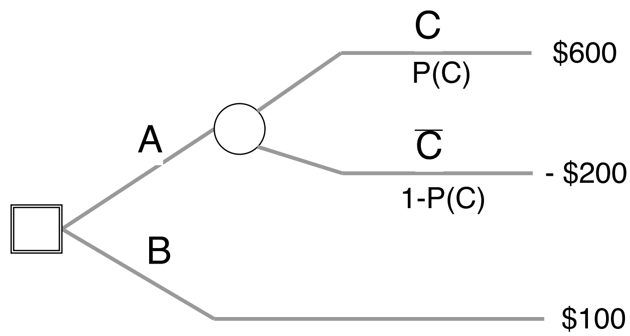
B. Write down the formula for value of information, plugging in numbers but do not calculate.

Value of information = Expected value with expert info - Expected value without

$$\text{Value of information} = \Big[(\$25)(0.4)(0.5) + (\$10)(0.6)(0.5) + (\$25)(0.4)(0.5) + (-\$5)(0.6)(0.5)\Big] - \Big[(\$25)(0.4) + (-\$5)(0.6)\Big]$$

C. How is minimum expected regret related to the value of information? (brief answer)

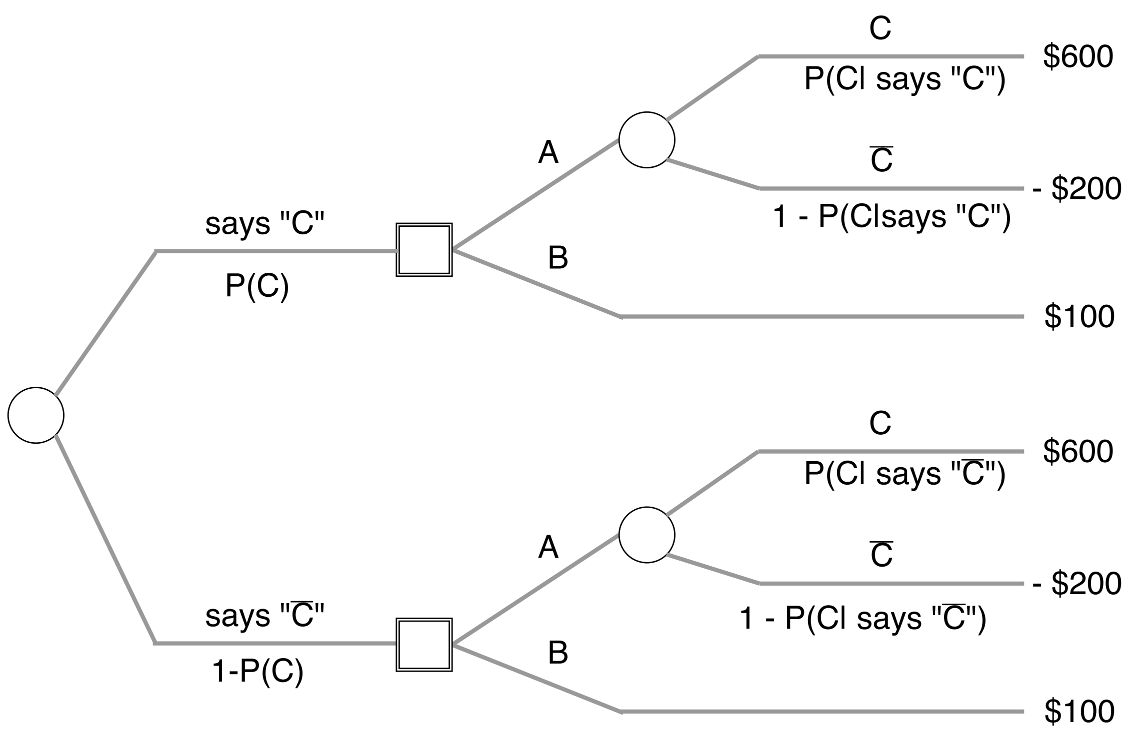
The minimum expected regret is equal to the value of (perfect) information. (at least in simple cases like this when decisions do not affect uncertainty nodes)



D. Consider the decision tree above. Suppose you consult an imperfect expert who will tell you his prediction for the outcome of the uncertainty node. His accuracy is

$$P(\text{ says } C \mid C\ )$$
$$P(\text{ says } \overline{C} \mid \overline{C}\ )$$

Draw the new decision tree where you use this information. Fully label the tree.



E. For each probability you need in the new tree, write them in terms of probabilities ($P(\cdot)$, $P(\cdot|\cdot)$) you know.

$$\begin{aligned} P(\text{ says "}C\text{"}) &= P(\text{ says "}C\text{"}, C) + P(\text{ says "}C\text{"}, \overline{C}) \\ &= P(C)P(\text{ says "}C\text{"}|C) + P(\overline{C})P(\text{ says "}C\text{"}|\overline{C}) \end{aligned}$$

$$P(\text{ says "}\overline{C}\text{"}) = 1 - P(\text{ says "}C\text{"})$$

$$P(C|\text{ says "}C\text{"}) = \frac{P(C)P(\text{ says "}C\text{"}|C)}{P(\text{ says "}C\text{"})}$$

$$P(\overline{C}|\text{ says "}C\text{"}) = 1 - P(C|\text{ says "}C\text{"})$$

$$P(\overline{C}|\text{ says "}\overline{C}\text{"}) = \frac{P(\overline{C})P(\text{ says "}\overline{C}\text{"}|\overline{C})}{P(\text{ says "}\overline{C}\text{"})}$$

$$P(C|\text{ says "}\overline{C}\text{"}) = 1 - P(\overline{C}|\text{ says "}\overline{C}\text{"})$$

Rubric

Later questions depend on responses to earlier questions.

A. [10 points]

- Labels and objective values [5 points]
  - -1 (total) if wrote "C" and "D" instead of "says C" and "says D"
  - -0.5 for each missing/wrong label (events, alternatives, and probabilities)
  - -0.5 for each missing/wrong objective value
- Form [5 points]
  - -4 if first node not uncertainty about what expert says (except if copied the original decision tree and added an alternative "consult expert")
  - -3 if uncertainty for A after hear expert's statement
  - -4 for other major form errors
  - -0 if broke expert statement into two sequential uncertainties

B. [4 points]

- -0 if do not write a formula with terms, but do have formula with values

C. [2 points]

- -0 if say they give the same strategy

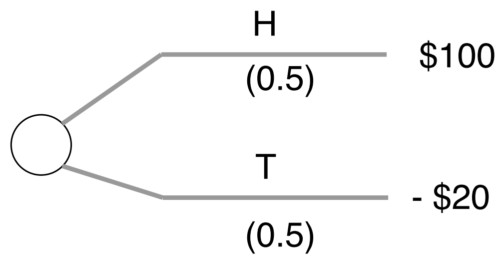
D. [10 points]

- Labels [3 points]
  - -0.5 for each missing/wrong label (events, alternatives, and probabilities)
- Objective values [2 points]
  - -0.5 for each missing/wrong objective value
- Form [5 points]
  - -4 if first node not uncertainty about what expert says (except if copied the original decision tree and added an alternative "consult expert")
  - -3 if no uncertainty for A after hear expert's statement
    * This could be if tree corresponds to perfect information or
    * there is uncertainty about C after expert information, but that occurs before decision node.

E. [9 points]

- -1.5 for each missing probability
- -1 for each incorrect formula (or if depends on unknown quantities, eg both "$P(\text{says } C) = 1 - P(\text{says } \overline{C})$" and "$P(\text{says } \overline{C}) = 1 - P(\text{says } C)$")

Problem 2. [35 points] This problem deals with utility.



A. Consider the gamble above. Your friend Jill has the opportunity to play this. You want to take her place. Jill says the least she would sell her place for is $10. What is Jill's risk premium for this gamble? (calculate the value)

$E[\text{ earnings }] = (\$100)(0.5) + (-\$20)(0.5) = \$50 - \$10 = \$40$

Certainty equivalent = $10

Risk Premium = $\$40 - \$10 = \$30$

B. What phrase describes her attitude toward risk?

Risk averse

C. Describe what a risk premium is in simple terms.

The risk premium is how much you are willing to "pay"(or give up the opportunity to earn) to avoid risk.

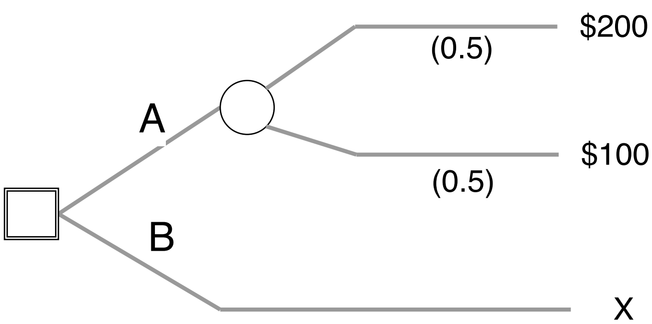
Your friend Jack is deciding between several job offers. He asks you to assess his utility curve $U(x)$ for salary.

D. You set the range to be from \$100k to \$200k. You will infer a normalized utility curve over this range. What utility values do the endpoints have?

$$U(\$100k) = 0 \qquad\qquad U(\$200k) = 1$$

You decide to infer his utility curve by partitioning the y-axis; that is, pick utility values and infer the corresponding salary.

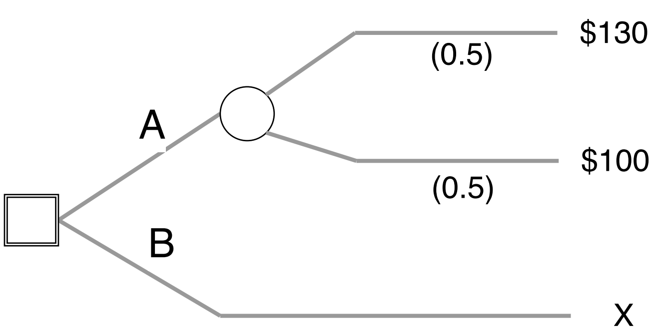
E. First set up a lottery to infer what salary x corresponds to $U(x) = \frac{1}{2}$. (draw the decision tree representing this lottery) Also show the formulas for why the lottery you designed yields $U(x) = \frac{1}{2}$.



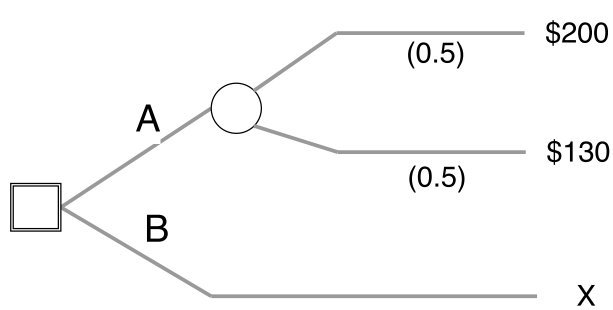
Instruct Jack to pick an X such that he would be indifferent between A and B. For that X,

$$\begin{aligned} U(X) &= \frac{1}{2}U(\$200k) + \frac{1}{2}U(\$100k) \\ &= \frac{1}{2}1 + \frac{1}{2}0 \\ &= \frac{1}{2} \end{aligned}$$

F. Suppose you conclude that for Jack, $U(130) = \frac{1}{2}$. Now set up two lotteries to infer what salary x corresponds to $U(x) = \frac{1}{4}$ and what salary x corresponds to $U(x) = \frac{3}{4}$.



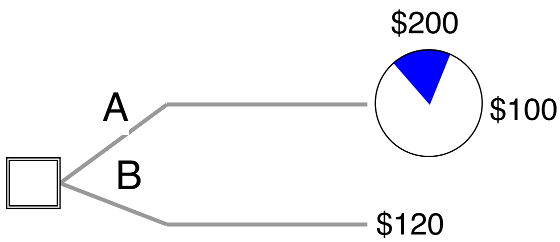
$$X = U^{-1}\left(\frac{1}{4}\right)$$



$$X = U^{-1}\left(\frac{3}{4}\right)$$

To check for consistency, you re-assess his utility curve by partitioning the x-axis; that is, picking salaries x and inferring the utility $U(x)$. (the range is still $100k to $200k)

G. Set up a lottery to infer Jack's utility for the salary $120k. What will you ask Jack to contemplate for this lottery? (draw the corresponding decision tree)



Instruct Jack to pick an area of shaded region (or the corresponding p) such that he would be indifferent between A and B.

H. Show how to calculate Jack's utility based on his response to the lottery.

Let p denote the proportion of shaded area for which Jack is indifferent. Indifference means the expected utilities match.

$$\begin{aligned} U(\$120k) &= pU(\$200k) + (1-p)U(\$100k) \\ &= p \times 1 + (1-p) \times 0 \\ &= p \end{aligned}$$

I. What are one pro and one con for using the method on this page (partition x-axis) compared to the method on the previous page (partition y-axis).

(these are some variations of the main pro/con)

Pro: Get values directly (for particular alternatives); easier to calculate with; don't need to interpolate; less work

Con: difficult/inaccurate for small and large p's; people have trouble understanding probabilities smaller/bigger than 1/2; values less accurate; more possibility of bias

Rubric

A. [4 points total]

- -0 if show formula with numbers only (don't write terms "risk premium" or "expected value")
- -1 if error due to calculation or plugging-in
- -2 if have negative
- -3 if wrong formula for other reasons or don't show any formulas but just write down "risk = $30"

B. [3 total]

- -2 if say risk-seeking (if premium positive; opposite if the calculated premium is negative)

C. [3 total]

- -2 if say opposite, how much pay to have risk
- -1 if state definition "it is the difference between expected value and certainty equivalent"
- -3 if something else

D. [2 points total; 1 each]

-

E. [6 points total]

- lottery [4 points]
  - -2 if don't have an uncertainty node
  - -1 total if have uncertainty node but not 1/2 probabilities
  - -1 total for wrong/missing objective values
  - -1 for not leaving certainty equivalent as variable
  - -1 total for writing utility as objective value (eg "$U(200k)$" instead of "$200k$")
  - if don't have certainty equivalence branch in the drawing
    * -1 if have formulas showing calculations using it
    * -2 if no evidence of it
- formulas [2 points]

F. [6 points total; 3 points each]

- -2 per lottery missing uncertainty node
- -1 per lottery with uncertainty node but without 1/2 probabilities
- -1.5 per lottery for incorrect/missing objective values
- -0.5 per lottery for not leaving certainty equivalent as variable
- -0.5 per lottery for writing utility as objective value (eg "$U(200k)$" instead of "$200k$")
- if don't have certainty equivalence branch in the drawing
  - -1 per lottery if have formulas showing calculations using it
  - -2 per lottery if no evidence of it

G. [6 points total]

- lottery [5 points]
  - risky alternative
    * -2 if don't have wheel of fortune or an uncertainty node with a variable p
    * -1.5 if do have but incorrect objective values

  - -2 if don't have $120k as a guaranteed alternative
  - -1 total for writing utility as objective value (eg "$U(200k)$" instead of "$200k$")
- instruction [1 point]

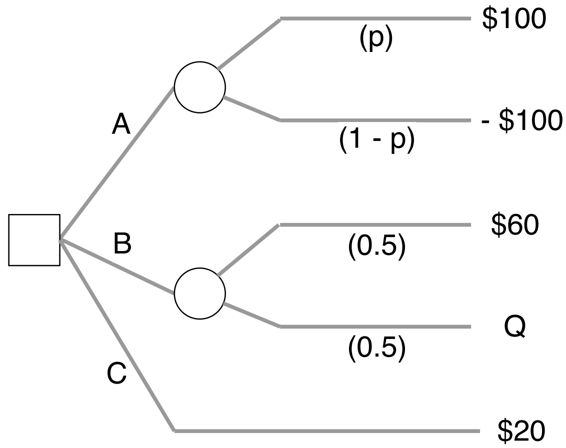
H. [3 points total]

- -1 if set up correct formula but do not solve for $U(\$120k)$

I. [2 points total; 1 point each]

-

Problem 3. [25 points] This question is about sensitivity analysis. Consider the following decision tree.



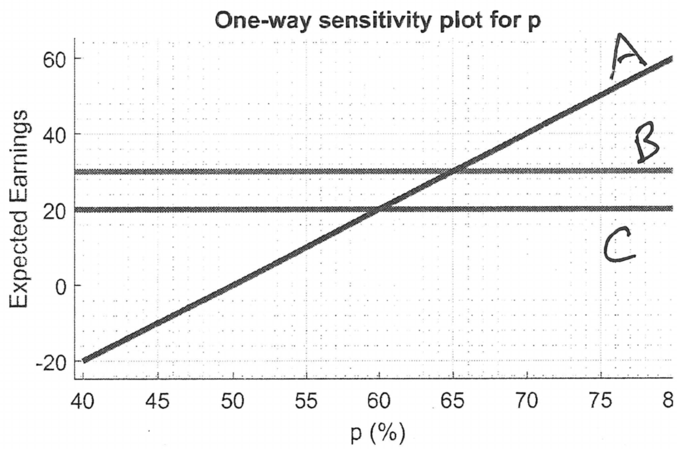
There are two parameters, p and Q, which are not known precisely.

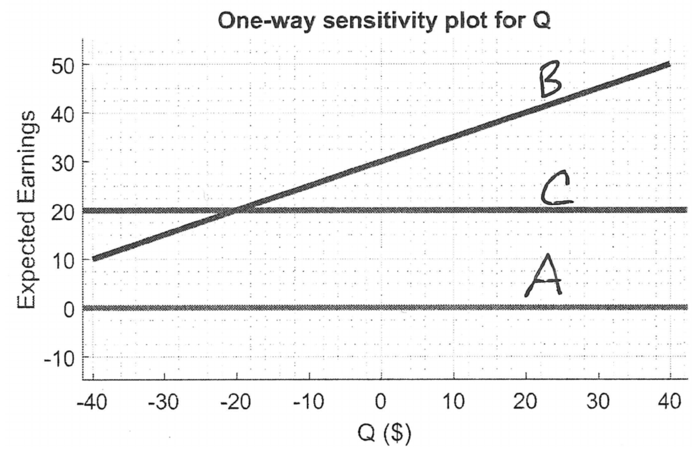
	low	base	high
p	40%	50%	80%
Q	-$40	$0	$40

Conduct sensitivity analysis. First, determine the expected earnings as a function of the variables p and Q. Then calculate the expected earnings for low, base case, and high values of p and Q:

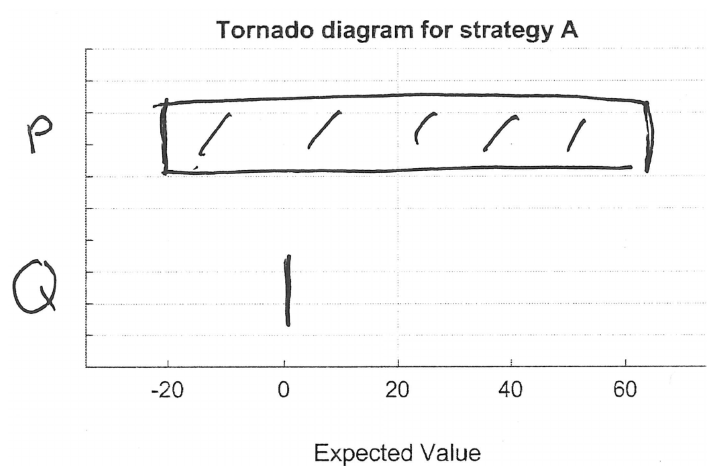
	formula (in terms of p and Q)	low	base	high
$\mathrm{E}\big[$ earnings of A $\big] =$	$p * 100 + (-100)(1-p) = 200p - 100$	-20	0	60
$\mathrm{E}\big[$ earnings of B $\big] =$	$0.5 * 60 + 0.5 * Q = 30 + \frac{1}{2}Q$	10	30	50
$\mathrm{E}\big[$ earnings of C $\big] =$	20	20	20	20

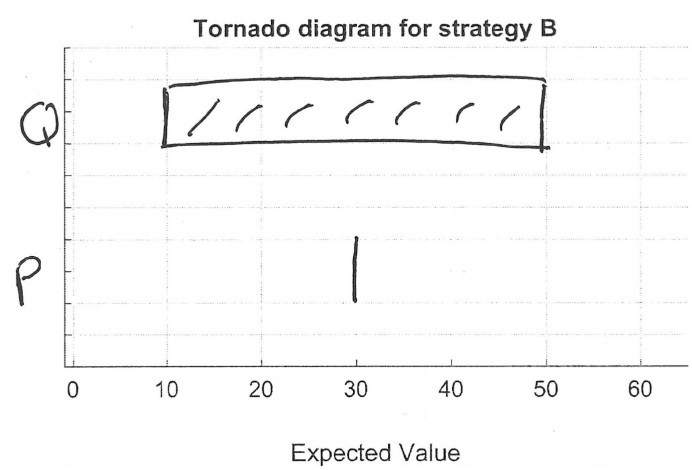
Draw the one-way sensitivity plots for p and Q. Label the curves.





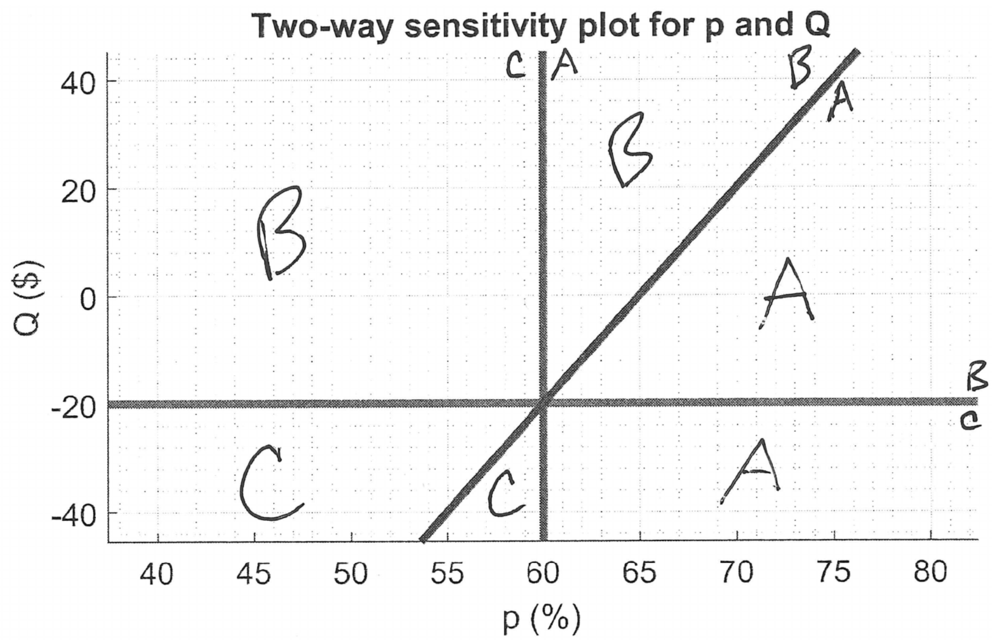
Draw the tornado diagrams for strategies A and B. Label the axes.





Draw the two-way sensitivity plot for p and Q.

- For each boundary denote which strategy is preferred on each side.
- For each region denote the preferred strategy.
- Show your work for solving for the boundary lines.



(Hint: for plotting the boundary line for A vs. B, solve for Q when p is 0.6 and 0.7)

$E[A] = E[C] \quad \Rightarrow$

$$\begin{aligned} 200p - 100 &= 20 \\ 200p &= 120 \\ p &= \frac{120}{200} = \frac{60}{100} = 0.6 \end{aligned}$$

$E[B] = E[C] \quad \Rightarrow$

$$\begin{aligned} 30 + \frac{1}{2}Q &= 20 \\ \frac{1}{2}Q &= -10 \\ Q &= -20 \end{aligned}$$

$E[A] = E[B] \quad \Rightarrow$

$$\begin{aligned} 200p - 100 &= 30 + \frac{1}{2}Q \\ 400p - 200 &= 60 + Q \\ Q &= 400p - 260 \end{aligned}$$

$$p = 0.6 \quad \Rightarrow \quad Q = 400\frac{60}{100} - 260 = 240 - 260 = -20$$

$$p = 0.7 \quad \Rightarrow \quad Q = 400\frac{70}{100} - 260 = 280 - 260 = 20$$

Rubric

To the extent errors do not over-simplify the work, each stage should be graded for correctness based on previous stage. For instance, if there's an error in the expectation formula, and the boundary line is correctly drawn for that (wrong) formula, then the only penalty should be for the initial formula.

- Expectations [4 points total]
  - -1 per formula error (with p and Q); they do not need to simplify
  - -0.25 per low/base-case/high error (max -1 total)

- One way plots [6 total]
  - -1 per plot label error
  - -1 per line drawn wrong (minor misalignments ok)

- Tornado plots [6 total]
  - -1 per y-axis label error (max -2 total)
  - -1 per rectangle wrong (minor misalignments ok)
  - -0 if wider rectangle on bottom

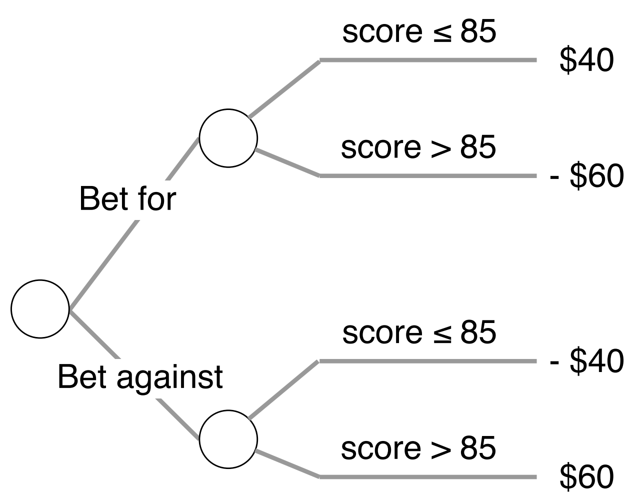
- Two way plot [9 points total]
  - -2 per boundary line formula error (1 point for setting up, 1 point for deriving)
  - -1 per boundary line drawing/labeling error (max -2 total)
  - -0.5 if label boundary line as $A = C$ or similar, so it is unclear from the line alone which side corresponds to which alternative (included in the max -2)
  - -0.5 per region labeling error (max -1; it is ok if they used dashed boundary lines to denote a common dominant alternative, eg one "B" written)

Problem 4. [20 points] Your friend Jake just finished a 72 hour ME 677 - Nonlinear Feedback Controller Design exam. Surprisingly, that was in-class, not a take-home.

He needs to decide whether he should sleep or study for his ME 680 - Bifurcations And Chaos exam, which starts at 10pm tonight.

He asks you to use all your IE 546 wizardry to help him decide. You begin by assessing his belief about his ME 680 exam score were he to not study.

A. He initially states that there's a 60% chance he won't get above an 85. Set up an initial symmetric lottery corresponding to this statement. Choose dollar amounts whose magnitudes add up to $100. Label the tree.

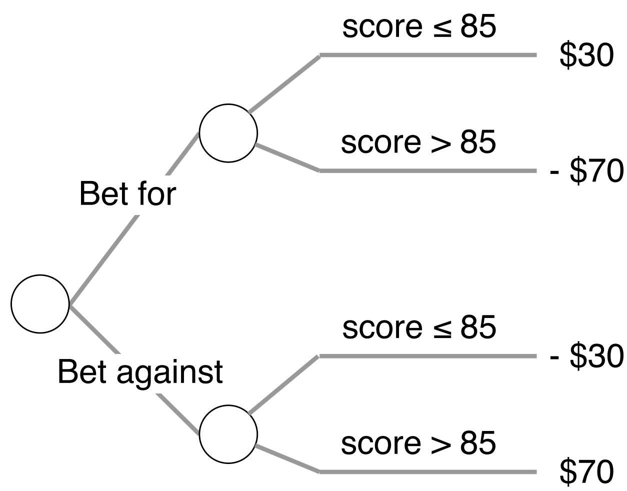


B. Suppose he says he prefers the top gamble. What does that mean about the probability 0.6 for a grade of 85? [Don't think about utility for this problem.]

It means his belief for $P($ score $\leq 85)$ is greater than 0.6, as he expects to make a profit off the top gamble. In simpler terms, 0.6 is too low.

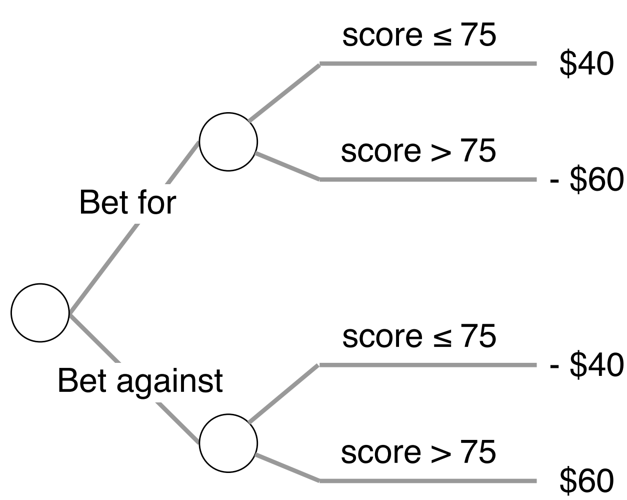
C. Suppose you want to find what his true belief is about getting at most an 85. Construct a new lottery in response to his preference. Label the tree. (the specific values don't matter, just how they differ from the lottery above)

Keep the events the same, but increase the probability for which the top and bottom branches have equal expected value. For instance, try 0.7 instead of 0.6 as in A.



D. Consider instead that you are trying to find the score that corresponds to a 60% belief (cumulative risk = 60%). Construct a new lottery in response to his preference. Label the tree. (the specific values don't matter, just how they differ from the original lottery)

Here we change events. Since his belief for $P($ score $\leq 85)$ is greater than 0.6, we need to try an event with a lower likelihood, such as score ≤ 75



Rubric

Later questions depend on responses to earlier questions.

A. [5 points]

- Labels [1 point]
  - -0.25 per missing label (-1 max)
- Objective values [4 total]
  - -3 if not a symmetric lottery
  - -1 if events incorrect (such as "$x = 85$")
  - if it is a symmetric lottery
    * -1 if magnitudes do not sum to 100 but otherwise correct
    * -1 if earnings correct for $p = 0.4$ (eg bet for is 60 and -40)
    * -2 if earnings don't correspond to $p = 0.6$ or 0.4.

B. [3 points] For this, it depends on how they drew the lottery.

- -1.5 if opposite conclusion

C. [6 points]

- Form [2 point]
  - -2 if not a proper symmetric lottery
  - -0.25 per missing label (up to -1 max)
- Changes [4 points]
  - -3 for (correctly) changing the event instead of the earnings
  - -2 for changing the earnings incorrectly (based on their conclusion in B.)
  - -2 for changing the earnings, but generically, such as with "X" and "Y"

D. [6 points]

- Form [2 point]
  - -2 if not a proper symmetric lottery
  - -0.25 per missing label (up to -1 max)
- Changes [4 points]
  - -3.5 for (correctly) changing the earnings instead of the event
  - -2 for changing the event incorrectly (based on their conclusion in B.)
  - -2 for changing the events, but generically, such as with "X" and "Y"

Scratch

Name (if detach but put work here): ____________________________

Scratch

Name (if detach but put work here): ____________________________

Scratch

Name (if detach but put work here): ______________________

Scratch Name (if detach but put work here): ______________________