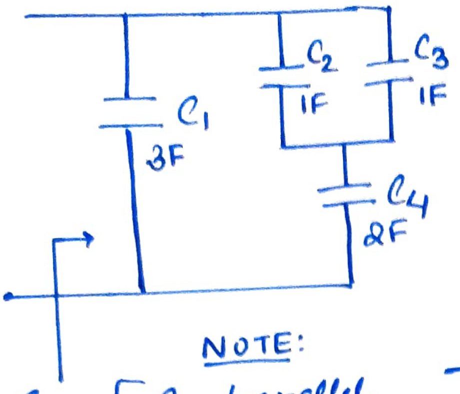
Exam 2- Version 1 Solutions:

1.)



'||' → parallel
'+' → series

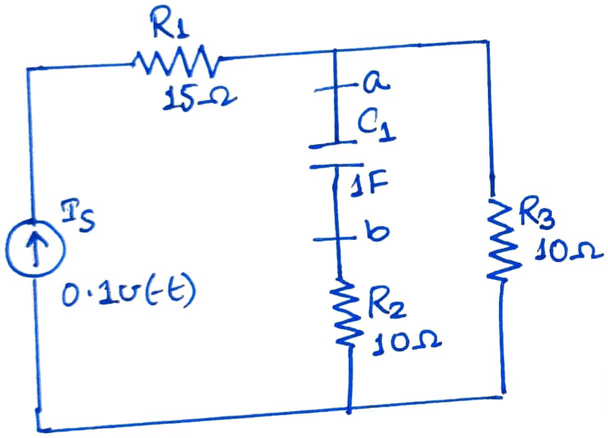
$$C_{eq} = [(C_2 \| C_3) + C_4] \| C_1$$
$$= [(1F \| 1F) + 2F] \| 3F$$
$$= [2F + 2F] \| 3F$$
$$= 1F \| 3F = 4F$$

$\boxed{C_{eq} = 4F}$ (Ans)

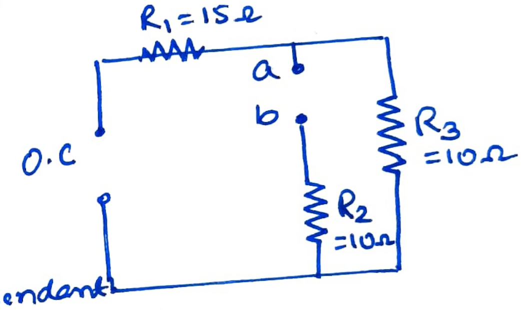
NOTE:

In parallel,
$$C_{eq_1} = C_1 + C_2$$
In series;
$$C_{eq_2} = \frac{C_1 C_2}{C_1 + C_2}$$

2.)



Across port ab (capacitor C):



Independant Current sources are treated as open sources.

$\therefore$ $R_{eq}\Big|_{\text{across port } a,b} = R_2 + R_3 = 20\Omega$

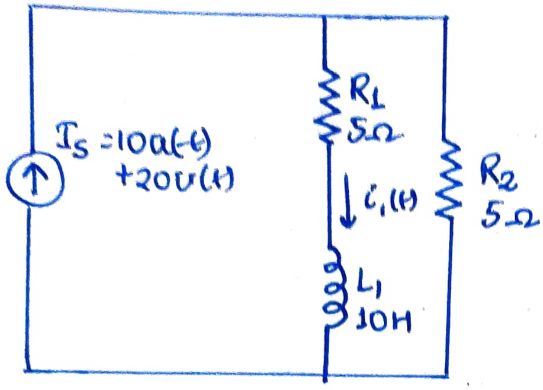
$\therefore$ Time constant $(\tau) = C \cdot R_{eq}$

$\therefore \tau = 1F \times 20\Omega$

$\boxed{\tau = 20s}$

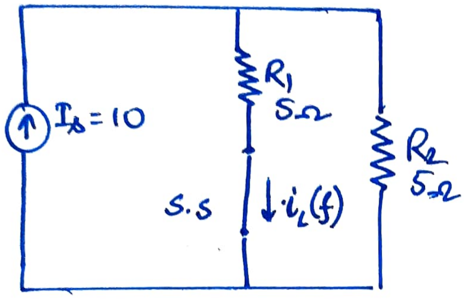
Ans

3.)



$$I_s = \begin{cases} 10 & ; t<0 \\ 20 & ; t>0 \end{cases}$$

<u>$t<0$:</u>



Inductor behaves as short circuit in steady state (s.s)

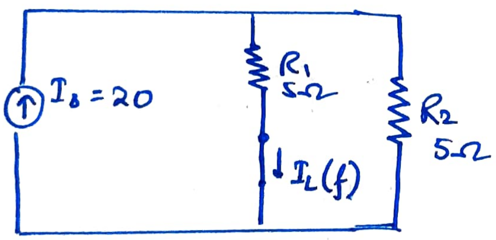
$i_L(\text{initial}) = 0$

$i_L(\text{final}) = \left(\frac{R_2}{R_1+R_2}\right) \times I_s$ [$\because$ current division]

$= \underline{5A}$

$\therefore i_L(0^-) = 5A$

<u>$t>0$:</u>



$i_L(\text{initial}) = i_L(0^+) = i_L(0^-) = 5A$

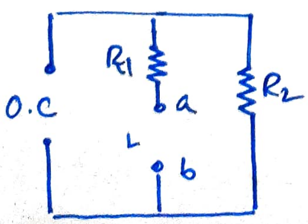
$i_L(\text{final}) = \left(\frac{R_2}{R_1+R_2}\right) \times I_s$

$= \frac{5}{10} \times 20$

$= \underline{10}\,A$

$\tau = L/R_{eq}$

$= \frac{L}{R_1+R_2} = \frac{10}{10} = 1s$



$\therefore i_L(t) = i_L(f) + (i_L(0^+) - i_L(f))e^{-\frac{(t-0)}{\tau}}$

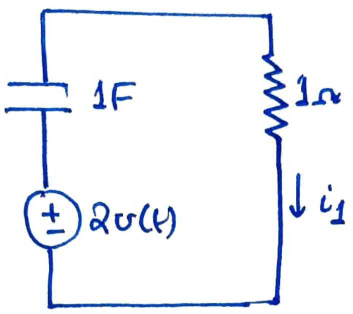
$= 10 + (5-10)e^{-t/1}$

$= 10 - 5e^{-t}$

$$\boxed{i_L(t) = 10 - 5\exp(-t) \quad t>0}$$

(Ans)

4)



$t>0$:

For capacitor C:

$V_c(0) = 0V$

$V_c(\text{final}) = 2V$

Capacitor reach steady state and behaves as Open circuit

$\tau = C \cdot R_{eq}$
$= 1F \times 1\Omega$
$= 1s$

$\therefore V_c(t) = 2 + (0-2)e^{-\frac{t-0}{1}}$
$= 2 - 2e^{-t}$

Now,

$\therefore \quad i_1(t) = C\frac{dV_c(t)}{dt} = \frac{d(2-2e^{-t})}{dt}$

$\therefore i_1(t) = 2e^{-t}$

$\boxed{i_1(t) = 2\exp(-t)_A}$ (Ans)

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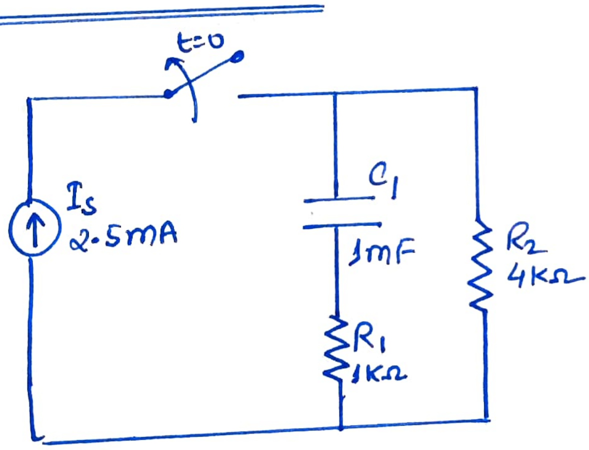
5)



$t<0$:

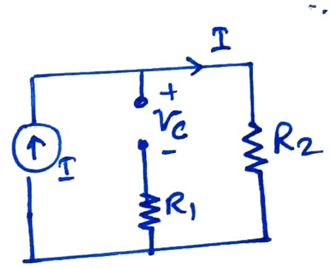
Capacitor is charged by current source:

$V_c(\text{initial}) = 0V$

$V_c(\text{final}) = I \times R_2 = 2.5mA \times 4k\Omega$
$= 10V$

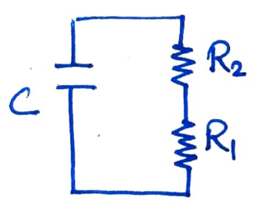
Capacitor reached S.S.

$\therefore V_c(0^-) = 10V$ --- (i)



$t>0$:

Capacitor starts to discharge:



$\tau = C \cdot R_{eq}$
$= 1mF(1+4)k\Omega$
$= 5s$

[$\because$ From (i)]

$V_c(0^+) = V_c(0^-) = 10V$

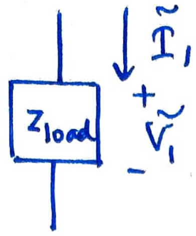
$V_c(\text{final}) = 0V$ (untill capacitor completely discharged)

$\therefore$ Energy delivered for $t>0$:

$\Delta E = \left|\frac{1}{2}CV_f^2 - \frac{1}{2}CV(0^+)^2\right|$
$= \frac{1}{2} 1mF \left|10^2 - 0^2\right|$

$\boxed{E = 50mJ}$ (Ans)

6.)



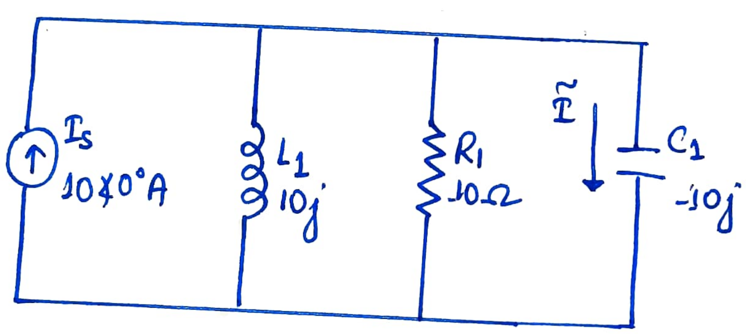
$$i_1(t) = -2\sin(120\pi t)$$
$$= -2\cos(120\pi t - 90^\circ)$$
$$\therefore \tilde{I}_1 = -2\angle -90^\circ$$

$$v_1(t) = 2\cos(120\pi t)$$
$$\tilde{V}_1 = 2\angle 0^\circ$$

$$\therefore Z_{load} = \frac{\tilde{V}_1}{\tilde{I}_1} = \frac{2\angle 0^\circ}{-2\angle -90^\circ} = -1\angle +90^\circ$$
$$= -j\Omega$$

$$\therefore \boxed{Z_{load} = -j\Omega} \quad \text{(Ans)}$$

7.)



$\because \omega = 120\pi$ rad/s

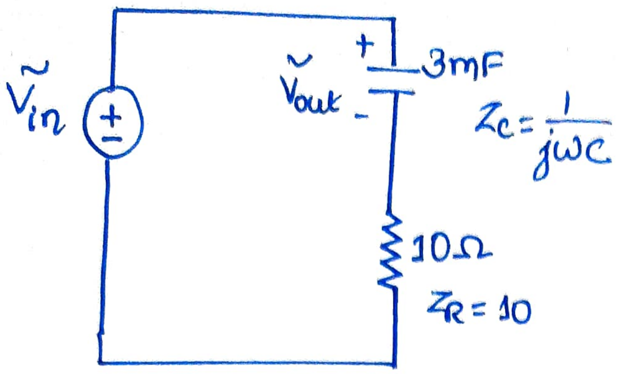
$$Y_{eq} = Y_L + Y_R + Y_C$$
$$= \frac{1}{j10} + \frac{1}{10} + \frac{1}{-10j}$$
$$= \left(\frac{1}{10}\right)$$

Applying current division in parallel combination:

$$Y_{eq} \cdot I_s = \tilde{I} \cdot Y_c$$
$$\text{or, } \tilde{I}\left(\frac{-1}{10j}\right) = \left(\frac{1}{10}\right) \cdot 10\angle 0^\circ$$
$$\text{or, } \tilde{I} \cdot (-0.1\angle 90^\circ) = 1\angle 0^\circ$$
$$\therefore \tilde{I} = -10\angle -90^\circ \quad [\because \omega = 120\pi]$$
$$= -10\cos(120\pi t - 90^\circ)$$
$$\boxed{\tilde{I} = -10\sin(120\pi t)}$$

(Ans)

8.)



From voltage division:

$$\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{Z_C}{Z_R + Z_C}\right|$$

Or, $$\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{1/j\omega C}{10 + \frac{1}{j\omega C}}\right| = \left|\frac{1}{10 j\omega C + 1}\right|$$

Given,

$$\left|\frac{V_{out}}{V_{in}}\right| \geq 0.8$$

$\therefore$ $$\frac{1}{(1 + \omega^2 \times (10 \times 3 \times 10^{-3})^2)^{1/2}} \geq 0.8$$

Or, $$\frac{1}{(1 + 9 \times 10^{-4} \omega^2)^{1/2}} \geq 0.8$$

Or, $$(1 + 9 \times 10^{-4} \omega^2) \leq \left(\frac{1}{0.8}\right)^2$$

Or, $$9 \times 10^{-4} \omega^2 \leq \frac{25}{16} - 1$$

Or, $$\cancel{9} \times 10^{-4} \omega^2 \leq \cancel{9}/16$$

Or, $$\omega^2 \leq \frac{10^4}{16}$$

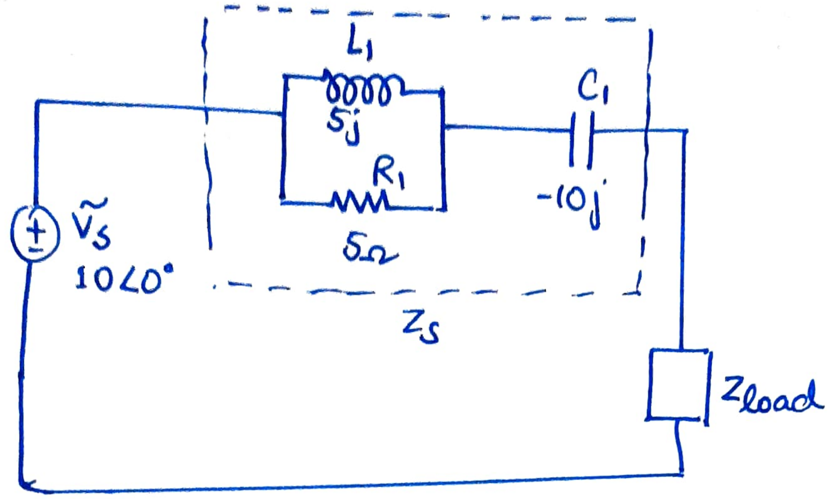
Or, $$\omega \leq \frac{10^2}{4}$$

Or, $$\omega \leq 25 \text{ rad/s}$$

$\therefore$ $\boxed{\text{Maximum value of } \omega = 25 \text{ rad/s}}$

(Ans)

9)



$$Z_S = (Z_L \parallel Z_R) + Z_C$$

$$= \left(\frac{5 \times 5j}{5+5j}\right) - 10j$$

$$= \frac{5j}{1+j} - 10j$$

$$= \cancel{10}\ \frac{5j(1-j)}{1-j^2} - 10j$$

$$= \frac{5j - 5j^2}{1+1} - 10j$$

$$= \frac{5}{2} + \left(\frac{5}{2} - 10\right)j$$

$$= \frac{5}{2} - 7.5j$$

For, maximum power,

$$Z_{Load} = Z_S^*$$

$$= \frac{5}{2} + (7.5j)$$

∴ Maximum power:

delivered by the source: $P_{max}^{(s)} = \frac{V_S^2}{Re[Z_S + Z_{Load}]^2} \times R_{Load}$

$$= \frac{V_S^2}{(R_S + R_L)^2} \times R_L$$

$[\because R_S = R_L]$ $= \frac{V_S^2}{4R_L^{\cancel{2}}} \times \cancel{R_L}$

$$= \frac{V_S^2}{4R_L}$$

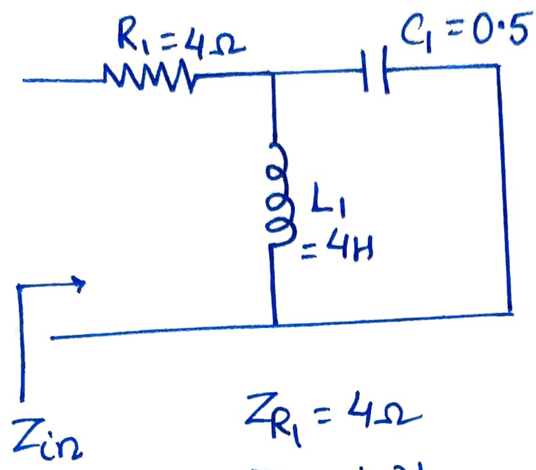
$$= \frac{100}{4 \times 5/2} = 10W$$

∴ Maximum Power delivered across Zload $= 10/2$ W

$= 5W$

∴ $\boxed{P_L = 5W}$ (Ans)

10)



$\omega = 1\ rad/s$

$Z_{R_1} = 4\Omega$

$Z_{L_1} = j\omega L_1$
$= j4\Omega$

$Z_{C_1} = \frac{-j}{\omega C} = \frac{-j}{0.5}$
$= -2j\ \Omega$

$Z_{in} = (Z_{C_1} \| Z_{L_1}) + Z_{R_1}$

$= \left(\frac{Z_{C_1} \times Z_{L_1}}{Z_{C_1} + Z_{L_1}}\right) + Z_{R_1}$

$= \left(\frac{(4j)(-2j)}{4j - 2j}\right) + 4$

$= \frac{8}{2j} + 4$

$= -4j + 4$

$= (4 - 4j)\ \Omega$

$\therefore \boxed{Z_{in} = (4-4j)\ \Omega}$ (Ans)