ECE 20001 Fall 2019

Final Exam

Last Name: Solution? (Not official)
(print)

First Name: Student
(print)

Student ID: ____________________

Section: (circle appropriate one)

Section 0002	9:30	Peroulis/Mayer
Section 0003	3:30	Michelusi
Section 0004	4:30	Upadhyaya
Section 0005	11:30	Peleato-Inarrea
Section 0006	2:30	Kildishev

I have neither given nor received unauthorized assistance on this exam.

____
(sign)

Instructions:

1. Adhere to the Purdue Honor Pledge. Sign the statement above before turning in your exam.
2. This is a closed-book, closed-note exam. No study materials should be visible or accessible during the exam. Use of a TI-30X IIS calculator is allowed.
3. An optical scan sheet is to be used to record your answers to the multiple-choice questions. Write your name, section number, and student identification number in the appropriate blocks on the optical scan sheet. Mark the corresponding letters and digits.
4. For each question, determine the answer and then select from among choices "①" through "⑩" the one that is closest to it. Mark the choice on this exam and on the optical scan sheet. Only the choice marked on the optical scan sheet will be scored. Your work to determine an answer may be reviewed as part of an academic integrity assurance process.
5. All questions are equally weighted but are not equally difficult – manage your time wisely.
6. If you need extra space for a question, use the bottom of the facing page. If you need more space than that, raise your hand and a proctor will provide an extra sheet of paper.
7. You have 120 minutes to complete the exam.
8. You must turn in (a) the optical scan sheet, (b) all pages of this exam, and (c) any extra sheet(s) provided by a proctor.

Potentially Useful Formulas

Response of a first-order network to step change of piecewise-constant input at t_0:

$$x(t) = x(\infty) + \big(x(t_0) - x(\infty)\big)e^{-(t-t_0)/\tau} \quad \text{for } t \geq t_0$$

Gauss's law in one dimension:

$$\frac{d\mathscr{E}}{dx} = \frac{\rho}{\varepsilon}$$

pn junction depletion region width:

$$W = \sqrt{\frac{2\varepsilon}{q}\frac{N_a + N_d}{N_a N_d}(V_{bi} - V_A)}$$

MOSFET I_D-V_{DS} characteristic:

$$I_D = \begin{cases} 0 & \text{for } V_{GS} < V_T \\ k\left[(V_{GS} - V_T)V_{DS} - \frac{1}{2}V_{DS}^2\right] & \text{for } V_{DS} \leq V_{GS} - V_T \\ k\left[\frac{1}{2}(V_{GS} - V_T)^2\right] & \text{for } V_{DS} \geq V_{GS} - V_T \end{cases}$$

The value of any necessary material or device parameter will be provided in the question.

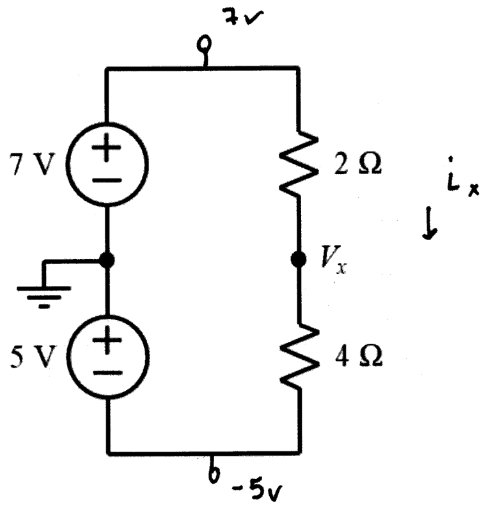
Learning Outcomes

i. An ability to analyze linear resistive circuits.
ii. An ability to analyze first-order linear circuits with sources and/or passive elements.
iii. An ability to analyze electronic circuits with diodes and transistors.

Question	LO	Points	Score
1	i	4	
2	i	4	
3	i	4	
4	i	4	
5	i	4	
6	i	4	
7	i	4	
8	i	4	
9	i	4	
10	i	4	
11	ii	4	
12	ii	4	
13	ii	4	
14	ii	4	
15	ii	4	
16	ii	4	
17	ii	4	
18	iii	4	
19	iii	4	
20	iii	4	
21	iii	4	
22	iii	4	
23	iii	4	
24	iii	4	
25	iii	4	

1. What is the value of V_x in volts?

① -7	⑥ 5
② -5	⑦ 6.333
③ 0	⑧ 7
④ 2	⑨ 8
● 3	⑩ 12



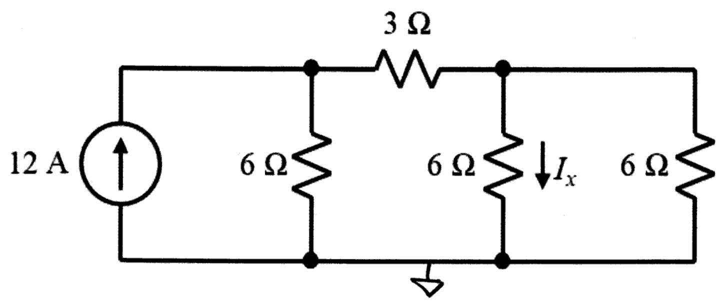
$7v - (-5v) = 12v$

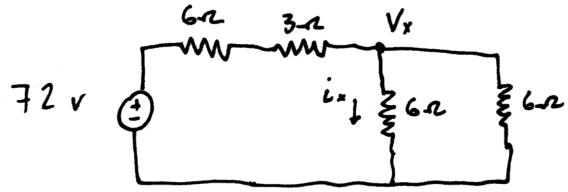
$\frac{12v}{6\Omega} = i_x = 2A$

$V_x = 7v - (2A)(2\Omega) = 3V$

2. What is the value of I_x in amperes?

① 0	⑥ 6
② 1	⑦ 8
③ 2	⑧ 9
● 3	⑨ 10
⑤ 4	⑩ 12



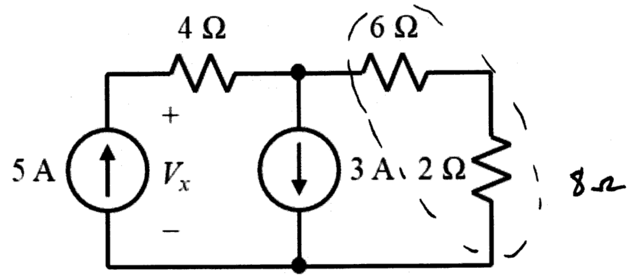


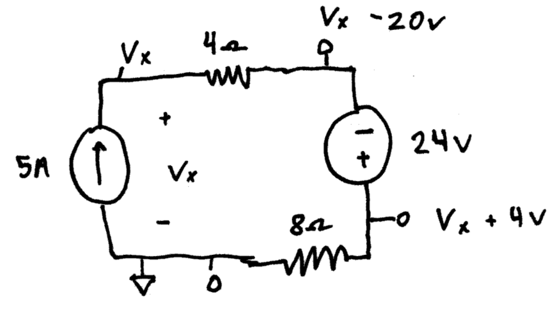
$$V_x = \left(\frac{6\Omega \| 6\Omega}{6\Omega \| 6\Omega + 9\Omega}\right)(72v) = \left(\frac{1}{4}\right)(72v) = 18v$$

$$\frac{18v}{6\Omega} = I_x = 3A$$

3. What is the value of V_x in volts?

① -36	⑥ 4
② -20	⑦ 16
③ -16	⑧ 20
④ -4	● 36
⑤ 0	⑩ 60





$$\frac{V_x + 4}{8} = 5A$$

40v - 4v = 36v

$V_x = 36$ V

4. What is the value of I_x in amperes?

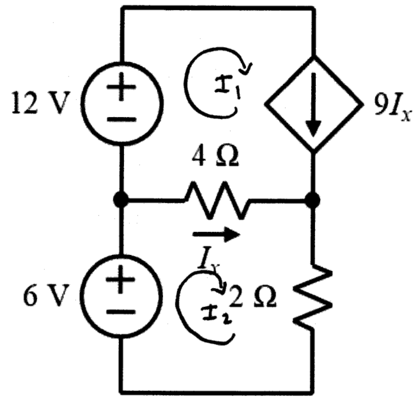
① -0.5 ● 0.25

② -0.25 ⑦ 0.5

③ -0.125 ⑧ 1

④ 0 ⑨ 1.5

⑤ 0.125 ⑩ 2



$I_2 - I_1 = I_x$

$-6 + 4(I_2 - I_1) + 2(I_2) = 0$

$I_2 - I_1 = I_x$

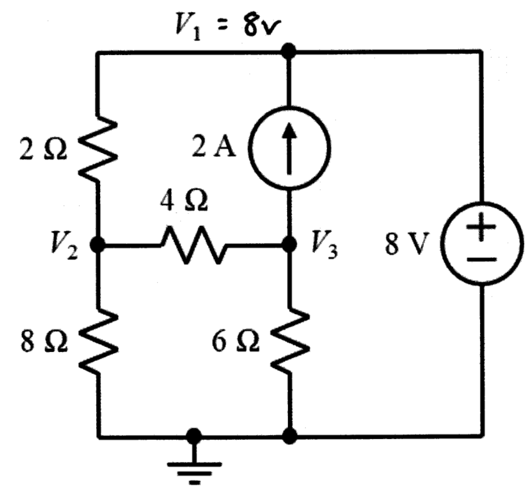
$I_1 = 9I_x$ $\quad$ $I_2 = 10 I_x$

$-6 + 4I_x + 2(10 I_x) = 0$

$24 I_x = 6$

$I_x = \frac{6}{24} = \frac{1}{4}$ A

For Questions 5 and 6 consider the following network model obtained via nodal analysis:



$$\begin{bmatrix} \frac{7}{8} & -\frac{1}{4} \\ -\frac{1}{4} & \text{B} \end{bmatrix}\begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \text{A} \\ -2 \end{bmatrix}$$

5. What is the value redacted by Box A?

① -8	⑥ 1
② -6	⑦ 2
③ -4	● 4
④ -2	⑨ 6
⑤ 0	⑩ 8

6. What is the value redacted by Box B?

① 0.1	● 0.417
② 0.125	⑦ 0.5
③ 0.167	⑧ 0.875
④ 0.25	⑨ 2.4
⑤ 0.375	⑩ 10

$$\frac{V_3 - V_2}{4} + 2 + \frac{V_3 - 0}{6} = 0$$

$$\hookrightarrow -2 = \frac{V_3}{4} - \frac{V_2}{4} + \frac{V_3}{6}$$

$$-24 = 3V_3 - 3V_2 + 2V_3$$

$$-24 = 5V_3 - 3V_2$$

$$\hookrightarrow -2 = \frac{5}{12} V_3 - \frac{1}{4} V_2$$

$$B = \frac{5}{12} = 0.416\bar{6} = \underline{0.417}$$

$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{4} + \frac{V_2}{8} = 0$$

$V_1 = 8v$

$$\frac{V_2}{2} - 4 + \frac{V_2}{4} - \frac{V_3}{4} + \frac{V_2}{8} = 0$$

$$\frac{3V_2}{4} + \frac{V_2}{8} - \frac{V_3}{4} = 4$$

$$\hookrightarrow \frac{7}{8} V_2 - \frac{1}{4} V_3 = 4 \rightarrow \underline{A = 4}$$

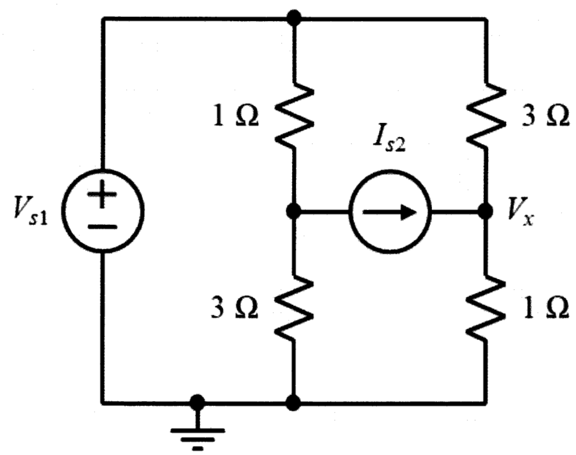
~~$9V_2 + 2V_2 - 3V_3 = 248$~~

~~$11V_2 - 3V_3 = 48$~~

~~$\frac{11}{12} V_2 - \frac{1}{4} V_3 = 24$~~

For Questions 7 and 8 express voltage V_x in the following form

$$V_x = a_1 V_{s1} + a_2 I_{s2}$$



7. What is the value of a_1?

① −1	● ¼
② −¾	⑦ ⅓
③ −½	⑧ ½
④ −⅓	⑨ ¾
⑤ −¼	⑩ 1

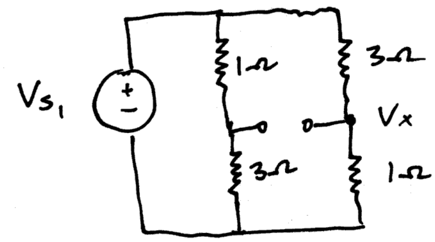
8. What is the value of a_2?

① −1	⑥ ¼
② −¾	⑦ ⅓
③ −½	● ½
④ −⅓	⑨ ¾
⑤ −¼	⑩ 1

$$V_x = \frac{1}{4} V_{s_1} + \frac{1}{2} I_{s_2}$$

Superposition

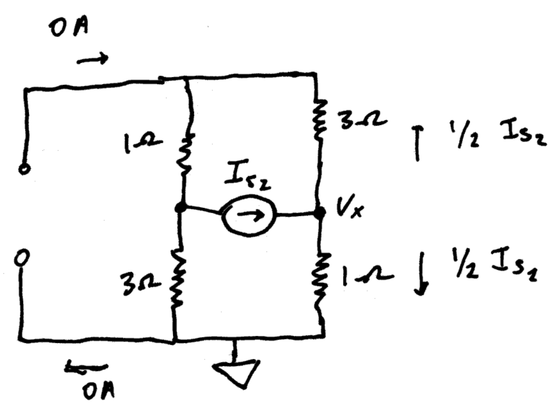
①



$$\left(\frac{1\Omega}{1\Omega + 3\Omega}\right)(V_{s_1}) = V_x \text{ due to } V_{s1}$$

$$\frac{1}{4} V_{s_1}$$

②

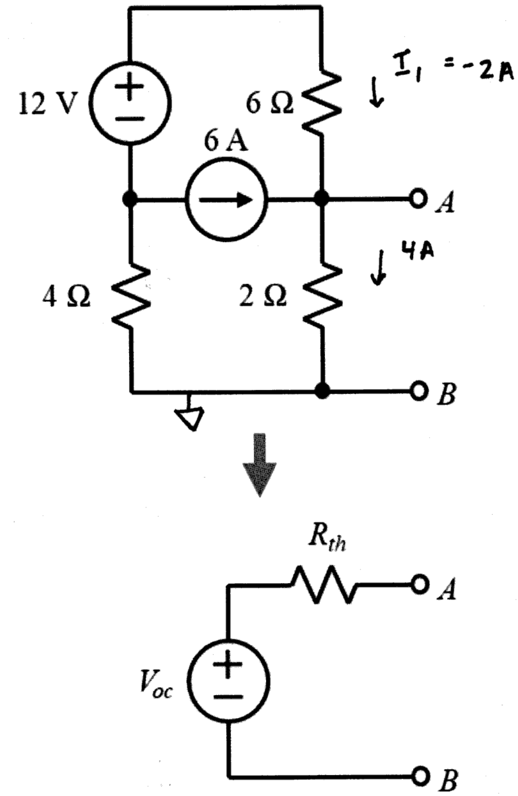


V_x due to $I_{s_2} = (1\Omega)\frac{1}{2} I_{s_2}$

Since each branch was equal resistance

3+1 = 1+3, the current splits evenly.

For Questions 9 and 10 consider the Thevenin equivalent network looking into terminal pair *A*-*B*.



9. What is the value of V_{oc} in volts?

① -8	⑥ 3
② -6	⑦ 4
③ -2	⑧ 6
④ 0	● 8
⑤ 2	⑩ 12

10. What is the value of R_{th} in ohms?

① 0	⑥ 4
② 1.091	⑦ 6
③ 1.333	⑧ 8
● 1.667	⑨ 10
⑤ 2	⑩ 12

KVL: $-12 + 6(I_1) + 2(6+I_1) + 4(6+I_1) = 0$

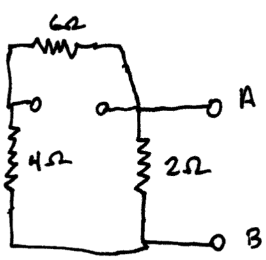
$12 = 6I_1 + 2I_1 + 4I_1 + 12 + 24$

$-24 = 12 I_1$

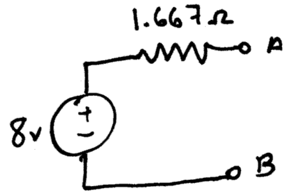
$I_1 = -2A$

$V_{oc} = (2\Omega)(4A) = 8V$

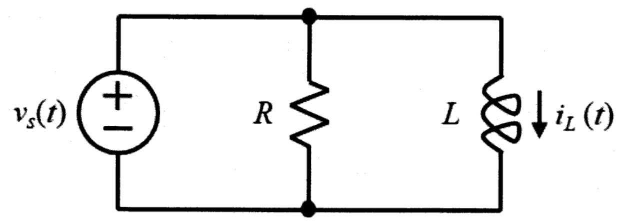
R_{th}:



$2\Omega \parallel 10\Omega = \frac{20}{12} = 1.667\Omega$



11. What is the value of $i_L(t)$ in amperes at $t = 0.75$ s if $R = 0.4\ \Omega$, $L = 0.5$ H, and $v_s(t) = 3u(t)$ V?



① 0	⑥ 3
② 1.125	⑦ 3.384
③ 1.354	⑧ 4.116
④ 1.646	● 4.5
⑤ 2.25	⑩ 7.5

$\tau = \frac{L}{R}$

$i_L(t) = i_\infty + (i_0 - i_\infty)\, e^{-\frac{t}{\tau}}$

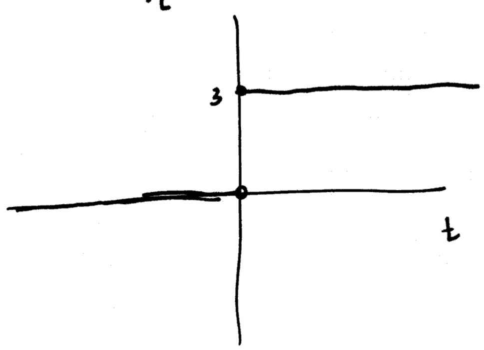
$i_\infty = \infty$ since $V_s(t)/0$ goes to infinity, thus there is a better model.

$V_L = L \frac{di_L}{dt}$

$V_L = V_s(t)$

$V_L = V_s$

$V_L = V_s$



$V_s\, dt = L\, di_L$

$\int \frac{V_s}{L}\, dt = \int di_L$

$\int \frac{3}{0.5}\, dt = \int di_L$

$6t + c = i_L(t)$

@ $t = 0$ $\quad i_L(t) = 0$

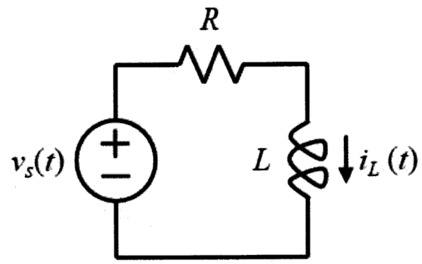
$0 = 0 + c$

$c = 0$

$6t = i_L(t)$

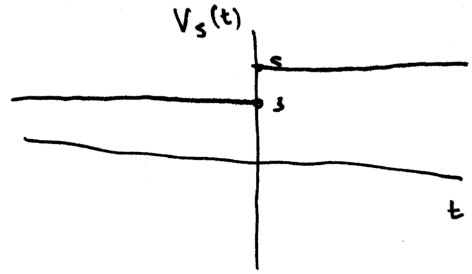
$i_L(0.75) = 6\,(0.75) = 4.5\text{A}$

For Questions 12 and 13 consider the inductor current response $i_L(t)$ when $R = 0.4\ \Omega$, $L = 0.5$ H, and $v_s(t) = 3u(-t) + 5u(t)$ V.



12. What is the value of $i_L(t)$ in amperes at $t = 0$?

① 0	⑥ 6
② 1.2	● 7.5
③ 1.5	⑧ 10
④ 3	⑨ 12.5
⑤ 5	⑩ 20



13. What is the value of $i_L(t)$ in amperes at $t = 0.6$ s?

① 0	⑥ 7.5
② 4.641	● 9.406
③ 4.765	⑧ 10.253
④ 4.9	⑨ 12.251
⑤ 5	⑩ 12.5

$i_L(0_-) = i_L(0) =$

$V_s(0_-) = 3\text{V}$

$\frac{3\text{V}}{0.4\Omega} = i_L(0) = 7.5\text{A}$

$i_L(\infty) = \frac{5\text{V}}{0.4\Omega} = \frac{50}{4} = 12.5\text{A}$

$i_L(t) = 12.5\text{A} - (12.5\text{A} - 7.5\text{A})\, e^{-\frac{t}{\tau}}$ $\qquad \tau = \frac{L}{R} = \frac{0.5}{0.4} = 1.25$

$i_L(0.6) = 12.5 - (5)\, e^{-\frac{0.6}{1.25}}$

$i_L(0.6) = 9.406\text{A}$

14. What is the expression for $i_x(t)$?

① 0

② 1.667

③ $0.527\cos(2t - 41.6°)$

④ $0.527\cos(2t + 41.6°)$

⑤ $1.581\cos(2t - 132°)$

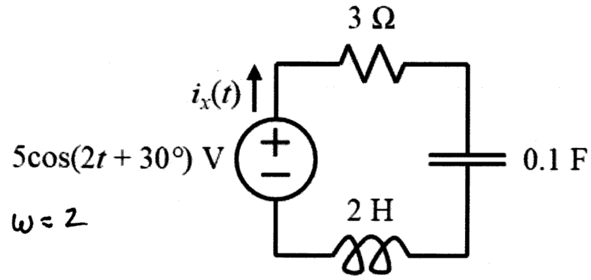
⑥ $1.581\cos(2t - 48.4°)$

⑦ $1.581\cos(2t - 18.4°)$

⑧ $1.581\cos(2t + 18.4°)$

● $1.581\cos(2t + 48.4°)$

⑩ $1.667\cos(2t + 30°)$

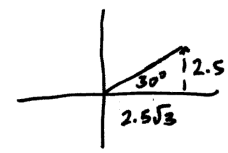


$\omega = 2$

$Z_L = j\omega L = 4j\,\Omega$

$Z_C = \frac{1}{j\omega C} = -\frac{1}{0.2} j\,\Omega = -5j\,\Omega$

$V_s(t) \Rightarrow \tilde{V}_s = 5 \angle 30° =$



$\tilde{V}_s = 2.5\sqrt{3} + 2.5j$ V

$V = IZ$

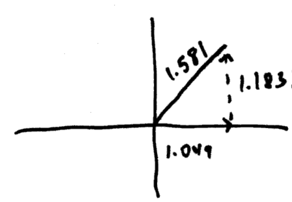
$Z_{circuit} = 3\,\Omega + 4j\,\Omega - 5j\,\Omega = 3 - j\,\Omega$

$V_{circuit} = \tilde{V}_s = 2.5\sqrt{3} + 2.5j$ V

$\frac{\tilde{V}_s}{Z_{circuit}} = \tilde{I}_x$

$\frac{2.5\sqrt{3} + 2.5j}{3 - j} = \tilde{I}_x = 1.049 + 1.183j$ A

↓

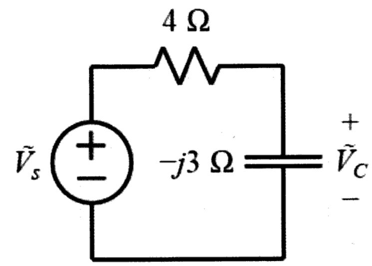


$|\tilde{I}_x| = 1.581$

$\phi = \arctan\left(\frac{1.183}{1.049}\right) = 48.4°$

$I_x(t) = 1.581\cos(2t + 48.4°)$

15. What is the <u>real part</u> of voltage phasor $\tilde{V}_C$ if $\tilde{V}_s = 10\angle 30°$ V?



① -0.718	⑥ 5.196
② 0.718	● 5.518
③ 3.143	⑧ 6
④ 3.6	⑨ 6.4
⑤ 3.712	⑩ 7.943

$\hat{V}_s = 10 < 30° =$

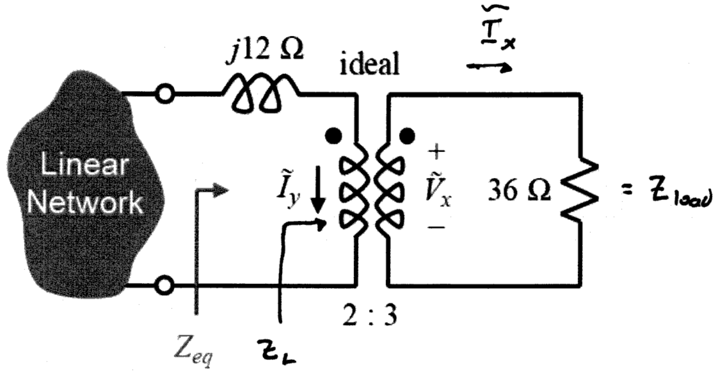
$\hat{V}_s = 5\sqrt{3} + 5j \ \text{V}$

$$\hat{V}_c = \left(\frac{-3j\,\Omega}{4-3j\,\Omega}\right)\hat{V}_s$$

$$\hat{V}_c = \frac{(5\sqrt{3}+5j)(-3j)}{(4-3j)} = 5.518 - 2.357j \ \text{V}$$

Real Part = 5.518

For Questions 16 and 17 consider the network shown to the right.



16. What is the magnitude of Z_{eq} in ohms?

① 0	⑥ 37.9
② 12	⑦ 45
● 20	⑧ 48
④ 26.8	⑨ 81.9
⑤ 36	⑩ ∞

17. What is the real part of $\tilde{I}_y$ in amperes if $\tilde{V}_x = 200\angle{-30°}$ V?

① -10.825	⑥ 3.208
② -8.333	⑦ 3.928
③ -7.217	⑧ 4.811
④ -4.811	● 7.217
⑤ -3.208	⑩ 8.333

$\frac{3}{2} = n$

$Z_L = \frac{Z_{load}}{n^2}$

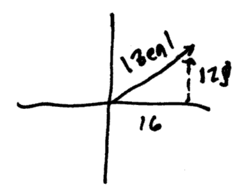
$Z_{load} = 36\ \Omega$

$n^2 = \left(\frac{3}{2}\right)^2 = 2.25$

$Z_L = \frac{36\ \Omega}{2.25} = 16\ \Omega$

$Z_{eq} = 12j + 16\ \Omega$

$|Z_{eq}| = \sqrt{12^2 + 16^2}$



$|Z_{eq}| = \sqrt{400} = 20\ \Omega$

$\frac{I_x}{I_y} = \frac{1}{n}$

if $\tilde{V}_x = 200 \angle -30°$

$\tilde{I}_x = \frac{200\angle -30°}{36\ \Omega}$

$= \frac{50}{9}\angle -30°$

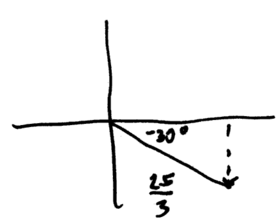
$\frac{\left(\frac{50}{9}\right)\angle -30°}{\tilde{I}_y} = \frac{2}{3}$

$\frac{3}{2}\left(\frac{50}{9}\right)\angle -30° = \tilde{I}_y$

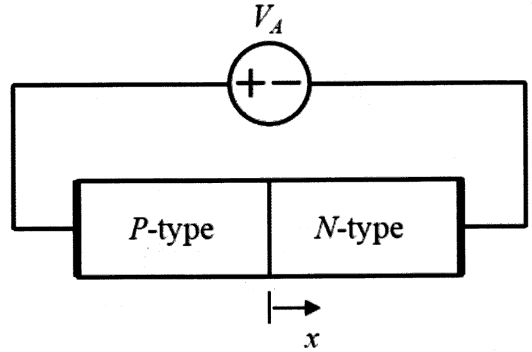
$\frac{25}{3}\angle -30° = \tilde{I}_y$

$\frac{25}{3}\cos(-30°) = \tilde{I}_y$ real comp.

$= 7.217$



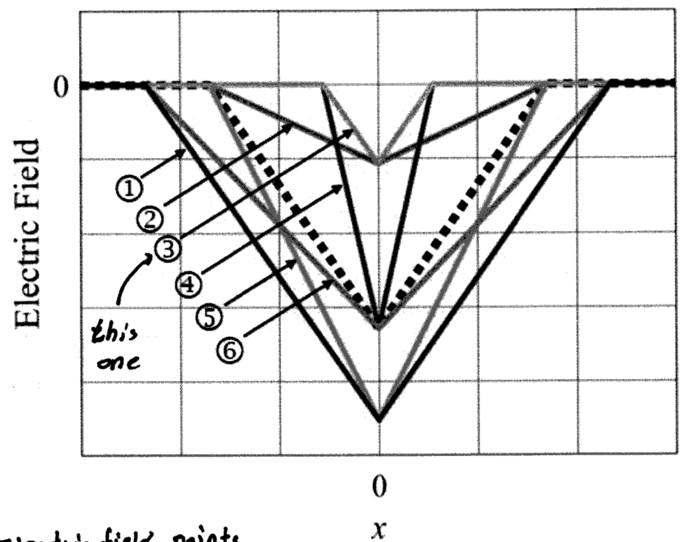
For Questions 18 and 19 assume that the *pn* junction is abrupt and symmetric ($N_a = N_d$). Also, assume that the junction is modeled using the depletion approximation. These assumptions are consistent with the majority of explanations/examples in the e-text, videos, and lectures.

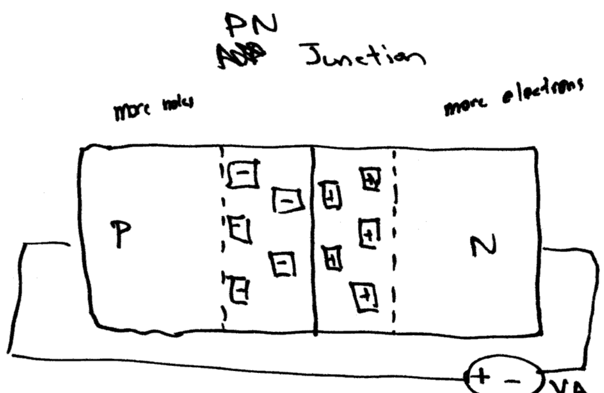


18. Which profile represents the electric field under forward bias if the dashed profile represents the electric field at equilibrium?

(3)

If the potential across the junction increases in its initial state, then depletion width increases, else it narrows.





Electric field points from + to −, so right to left

+++++ ← higher potential

- - ---

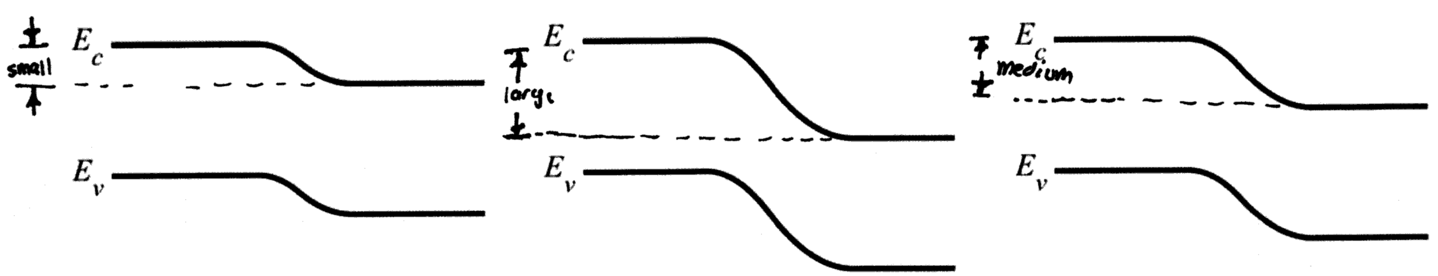
When $V_A > 0$ it weakens the electric field that is built in, so it weakens and depletion width gets smaller.

19. Which sequence of applied voltage conditions corresponds to the energy band diagrams shown below?

① $V_A < 0$, $V_A = 0$, $V_A > 0$
② $V_A < 0$, $V_A > 0$, $V_A = 0$
③ $V_A = 0$, $V_A < 0$, $V_A > 0$
④ $V_A = 0$, $V_A > 0$, $V_A < 0$
● $V_A > 0$, $V_A < 0$, $V_A = 0$
⑥ $V_A > 0$, $V_A = 0$, $V_A < 0$

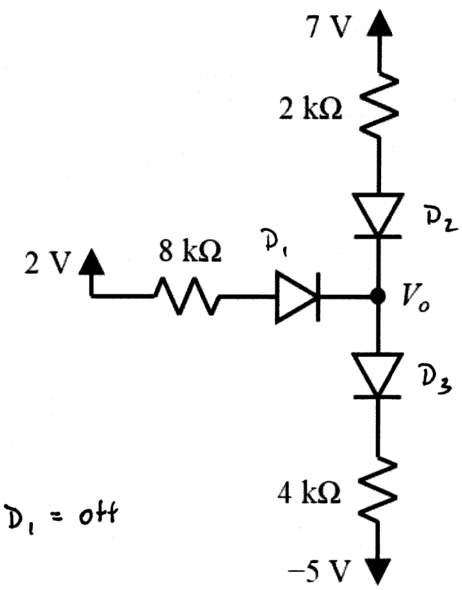
* Large bandgap means harder to move current → reverse bias ⇔ $V_A < 0$

* Small bandgap → The V_{bi} is reduced which happens when $V_A > 0$



20. What is the value of V_o in volts if the ideal diode model is used for each of the diodes?

① -5 ⑥ 2.667
② -2.667 ⑦ 2.857
③ 0 ● 3
④ 2 ⑨ 6
⑤ 2.5 ⑩ 7



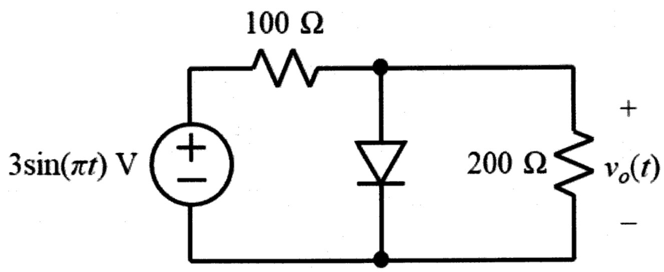
D_2 and D_3 are on and D_1 = off

$$V_o = -5V + \left(\frac{4k\Omega}{6k\Omega}\right)\left(7V - (-5V)\right) = -5V + 8V$$

$V_o = 3V$

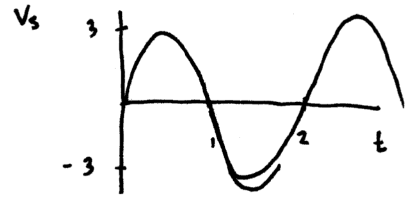
Since $V_o = 3V$, this makes D_1 turn off, verifying the assumption.

For Questions 21 and 22 use the ideal diode model.



21. What is the <u>maximum</u> value of $v_o(t)$ in volts?

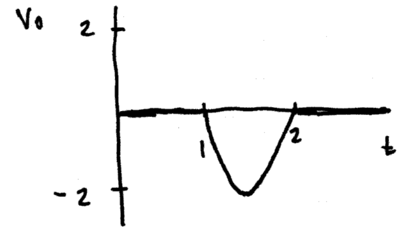
① -3	⑥ 1
② -2	⑦ 1.5
③ -1	⑧ 2
● 0	⑨ 2.121
⑤ 0.7	⑩ 3



Diode is on if $V_s > 0$, off is $V_s \leq 0$

22. What is the <u>average</u> value of $v_o(t)$ in volts?

① -3	⑥ 0.637
② -2	⑦ 0.955
③ -0.955	⑧ 2
● -0.637	⑨ 2.121
⑤ 0	⑩ 3



Max of $V_o(t) = 0V$

Average Value of $V_o(t)$: $\dfrac{\int_0^2 V_o(t)\,dt}{2}$

$$\int_0^2 V_o(t)\,dt = \int_0^1 0\,dt + \int_1^2 2\sin(\pi t)\,dt$$

$$= 0 + \int_1^2 2\sin(\pi t)\,dt =$$

$$= -\frac{2}{\pi}\cos(\pi t)\Big|_{t=1}^{t=2}$$

$$= -\frac{2}{\pi}\left(\cos(2\pi) - \cos(\pi)\right)$$

$$= -\frac{2}{\pi}\left(1 - (-1)\right) = -\frac{4}{\pi}$$

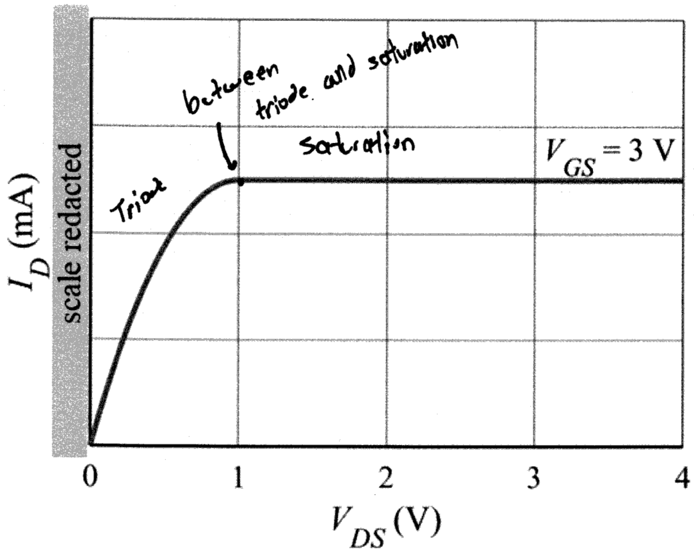
$$\overline{V_o(t)} = \frac{-\frac{4}{\pi}}{2} = -\frac{2}{\pi}$$

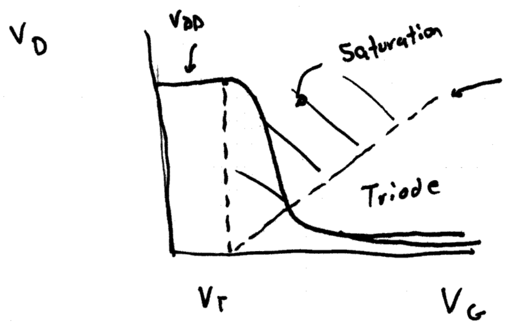
$$-\frac{2}{\pi} = -0.637$$

23. What is the value of the threshold voltage $\underline{V_T}$ in volts for a MOSFET with this $\overline{I_D}$-V_{DS} characteristic?

- ① -1
- ② 0
- ③ 0.5
- ● ~~1~~
- ⑤ 1.5
- ● 2
- ⑦ 2.5
- ⑧ 3
- ⑨ 4
- ⑩ 5

actual answer





(This line splits triode and saturation zones)

beginning of saturation region which happens w/ near constant I_D

↳ happens when on $V_g - V_T$ line where $V_D = V_G - V_T$

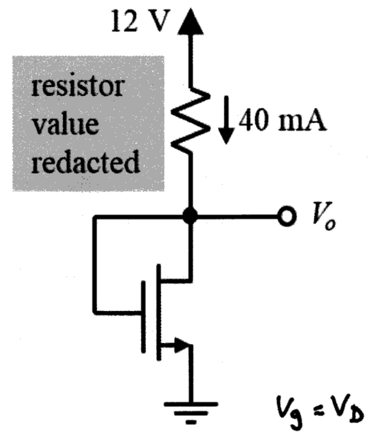
$$V_G - V_D = V_T$$

$$3V - 1V = V_T$$

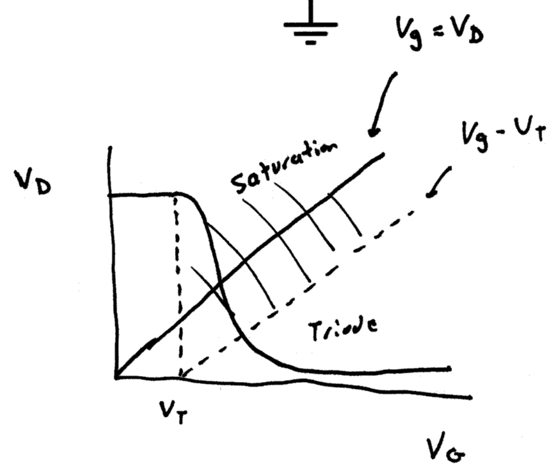
$$\boxed{V_T = 2V}$$

24. What is the value of V_o in volts if $V_T = 1$ V and $k = 5$ mA·V^{-2}?

① 0	● 5
② 1	⑦ 6
③ 2	⑧ 8
④ 3	⑨ 10
⑤ 4	⑩ 12



$V_g = V_o = V_D$



$V_g = V_D = V_o$

Voltage at gate = voltage at drain.

V_G

$$I_D \begin{cases} 0 & \text{if } V_g < V_T \\ k\left[(V_g - V_T)V_D - \frac{1}{2}(V_D)^2\right] & \text{if } V_D < V_g - V_T \leftarrow \text{triode} \\ \frac{k}{2}(V_g - V_T)^2 & \text{if } V_D > V_g - V_T > 0 \leftarrow \text{saturation} \end{cases}$$

if $V_g = V_D$, then $V_g - V_T$ must be less than V_D, meaning saturation region

* Since current flows, $V_g > V_T$

$\hookrightarrow I_D = \frac{k}{2}(V_g - V_T)^2$

~~$I_D = 40\text{mA} = \frac{5\text{ mA}}{V^2}\left((V_g - 1V)V_D - \frac{1}{2}(V_D)^2\right)$~~

$V_g = V_o$

~~$40\text{mA} = \frac{5\text{mA}}{V^2}\left((V_o - 1)(V_o) - \frac{1}{2}V_o^2\right)$~~

$40\text{mA} = \frac{5}{2}\frac{\text{mA}}{V^2}(V_o - 1)^2$

~~$8 = V_o^2 - \frac{V_o^2}{2} - V_o$~~

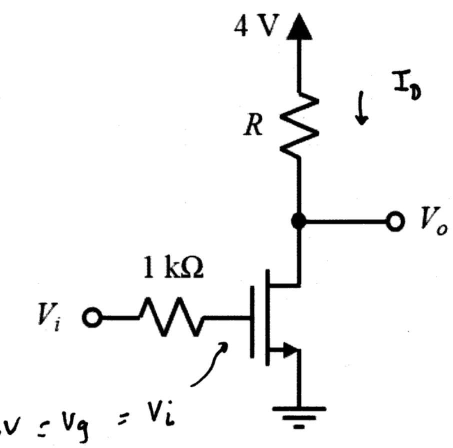
$\sqrt{16} = V_o - 1$

~~$0 = \frac{V_o^2}{2} - V_o - 8$~~ ~~$0 = V_o^2 - 2V_o - 16$~~

$5V = V_o$

25. What is the value of R in ohms if $V_T = 1$ V, $k = 2$ mA·V^{-2}, and $V_o = 0.5$ V when $V_i = 3$ V?

① 500	⑥ 1556
② 750	● 2000
③ 875	⑧ 2286
④ 1000	⑨ 4000
⑤ 1167	⑩ 5000



$3V = V_g = V_i$

$I_D = \frac{4V - V_o}{R}$

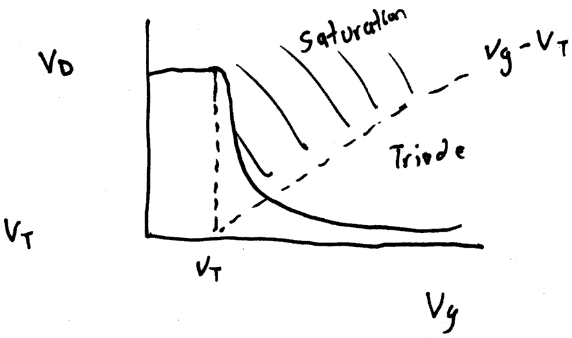
$I_D = \frac{4 - 0.5}{R}$

$V_g = 3V$

$V_o = V_D = 0.5V$

$V_g - V_T = 3 - 1 = 2V$

$2V > 0.5V$, thus $V_D < V_g - V_T$ → triode region



$I_D = k\left[(V_g - V_T)(V_D) - \frac{1}{2}(V_D)^2\right]$

since $V_d < V_g - V_T$

$V_g = 3V$

$V_o = 0.5V = V_D$

$V_T = 1V$ $\quad$ $V_D = 0.5V$

$k = \frac{2mA}{V^2}$

$I_D = \frac{2mA}{V^2}\left[(3-1)(0.5) - \frac{1}{2}(0.5)^2\right]$

$I_D = 2\left(\frac{3}{8}\right)$ mA $= 0.75$mA

$I_D = \frac{4 - 0.5}{R} = 0.75 \cdot 10^{-3}$ A

$R = 2000\ \Omega$